RESONANT FREQUENCIES OF DIELECTRIC LOADED WAVEGUIDE CAVITIES

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by

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Abstract

An accurate method for the computation of the resonant frequencies of dielectrically loaded circular cylindrical waveguide cavities is described. The method is suitable for the calculation of any resonant mode with high accuracy. Comparison of the calculation of the resonant frequencies of several cases agreed closely with measurements.

Introduction

The recent availability of low loss, temperature stable high dielectric constant materials[1] has generated increased interest in the utilization of such materials in several microwave components. One of the most interesting applications of dielectric resonators is in high quality dual mode bandpass filters[2] for satellite transponders. To design such high quality filters, it is important to be able to compute accurately the resonant frequency of practical resonator configurations. Although several workers presented approximate methods for the computation of the resonant frequencies of dielectric resonators excited in the fundamental $TE_{10\delta}$-mode,[3,4] taking into account mounting structures and enclosures, little attention has been given to other useful or spurious mode calculations. This paper describes a rigorous method for the computation of the resonant frequency of dielectric loaded circular waveguide cavities, excited in non-axially symmetric modes, taking into account effects of supporting structures and enclosures. The method can also be applied to construct mode charts which help determine other resonant frequencies of the same structure that may appear as spurious modes. Results of measurements on several resonators agree with the computations within a fraction of a percent.

Theory

The structure under consideration is shown in Figure 1. The cavity of radius b and length L has perfectly conducting walls. The high dielectric constant cylinder (ε_{r1}) has radius a and length ℓ, and is supported by low dielectric supports (ε_{r3}) on each side. The space $a < r < b$ is filled with a low dielectric constant material (ε_{r2}), which also serves as a support for the resonator.

To compute the resonant frequency, the structure is divided into three regions: A, B and C indicated Figure 1. In each of these regions the total fields are expressed in terms of the appropriate normal waveguide modes. The transverse fields are then matched at the boundaries at $z = \pm \ell/2$. This results in an infinite set of linear homogeneous equations. Resonant frequencies of the structure are determined by equating to zero the determinant of a truncated subset of these equations.

The normal modes in the end regions (A and C) are the usual TE and TM modes of the circular waveguide. Region B fields are hybrid modes with axial electric and magnetic fields. The angular variation of the modes, however, must be the same in all three regions. The transverse fields in each of the three regions A, B, and C are expressed as:

$$\bar{E}_A = \sum_i a_i \hat{e}_i \sinh \bar{\gamma}_i (z + L/2) \quad (1a)$$

$$\bar{H}_A = -\sum_i a_i \hat{h}_i \cosh \bar{\gamma}_i (z + L/2) \quad (1b)$$

$$\bar{E}_B = \sum_i \hat{E}_i (A_i e^{-\gamma_i z} + B_i e^{\gamma_i z}) \quad (2a)$$

$$\bar{H}_B = \sum_i \hat{H}_i (A_i e^{-\gamma_i z} - B_i e^{\gamma_i z}) \quad (2b)$$

$$\bar{E}_C = \sum_i b_i \hat{e}_i \sinh \gamma_i (z - L/2) \quad (3a)$$

$$\bar{H}_C = -\sum_i b_i \hat{h}_i \cosh \gamma_i (z - L/2) \quad (3b)$$

where

$\bar{\gamma}_i$, $\hat{e}_i$ and $\hat{h}_i$ are the propagation constants, transverse electric and magnetic fields of the normal modes (i.e., TE or TM in the circular guide, respectively, and

γ_i, $\hat{E}_i$ and $\hat{H}_i$ are the propagation constant, transverse electric and magnetic fields of the hybrid modes in the dielectric loaded guide, respectively.

For the hybrid modes, the propagation constants $\gamma_i = j\beta_i$ are solutions of the equation:

$$U - k_o^2 a^2 VW = 0 \quad (4)$$

where

$$U = \left[n\beta J_n (\xi_1 a) \left(\frac{1}{\xi_1^2} + \frac{1}{\zeta_2^2}\right) \right]^2$$

$$V = \left[\frac{J'_n(\xi_1 a)}{J_n(\xi_1 a)\,\xi_1} + \frac{P'_n(\zeta_2 a)}{\zeta_2}\right]$$

$$W = \frac{\varepsilon_{r1}}{\xi_1} + \left[\frac{\varepsilon_{r2}\, R'_n(\zeta_2 a)}{\zeta_2}\right]$$

$$\xi_1^2 = \varepsilon_{r_1} k_o^2 - \beta^2,\ \zeta_2^2 = \beta^2 - \varepsilon_{r_2} k_o^2$$

$$P'_n(\zeta_2 a) = \left[\frac{K'_n(\zeta_2 a)\, I'_n(\zeta_2 b) - K'_n(\zeta_2 b)\, I'_n(\zeta_2 a)}{K_n(\zeta_2 a)\, I'_n(\zeta_2 b) - K'_n(\zeta_2 b)\, I_n(\zeta_2 a)}\right]$$

$$R'_n(\zeta_2 a) = \left[\frac{K'_n(\zeta_2 a)\, I_n(\zeta_2 b) - K_n(\zeta_2 b)\, I'_n(\zeta_2 a)}{K_n(\zeta_2 a)\, I_n(\zeta_2 b) - K_n(\zeta_2 b)\, I_n(\zeta_2 a)}\right]$$

$$k_o^2 = \omega^2 \mu_o \varepsilon_o = \left(\frac{2\pi}{\lambda o}\right)^2$$

$J_n(.)$, $I_n(.)$ and $K_n(.)$ are the Bessel function, modified Bessel functions of first kind and second kind, respectively.

When the transverse electric and magnetic field components of equations (1) to (3) are equated, and orthogonality relations[5] of the normal (TE and TM) modes are used, the following set of homogeneous linear equations result:

$$a_j\, s_j = \sum_i \left(A_i e^{\gamma_i \ell/2} + B_i e^{-\gamma_i \ell/2}\right) \langle \hat{E}_i, \hat{e}_j \rangle \quad (5a)$$

$$-a_j\, c_j = \sum_i \left(A_i e^{\gamma_i \ell/2} - B_i e^{-\gamma_i \ell/2}\right) \langle \hat{H}_i, \hat{h}_j \rangle \quad (5b)$$

$$-b_j\, s_j = \sum_i \left(A_i e^{-\gamma_i \ell/2} + B_i e^{\gamma_i \ell/2}\right) \langle \hat{E}_i, \hat{e}_j \rangle \quad (5c)$$

$$-b_j\, c_j = \sum_i \left(A_i e^{-\gamma_i \ell/2} - B_i e^{\gamma_i \ell/2}\right) \langle \hat{H}_i, \hat{h}_j \rangle \quad (5d)$$

where

$$\langle \hat{E}_i, \hat{e}_j \rangle = \int_S (\hat{E}_i \cdot \hat{e}_j^{\,*})\, dS;\quad s_j = \sinh \bar{\gamma}_j \left(\frac{L-\ell}{2}\right)$$

$$\langle \hat{H}_i, \hat{h}_j \rangle = \int_S (\hat{H}_i \cdot \hat{h}_j^{\,*})\, dS;\quad c_j = \cosh \bar{\gamma}_j \left(\frac{L-\ell}{2}\right)$$

Determination of the resonant frequency of the structure is obtained by truncating at a certain number N of normal waveguide modes and hybrid modes, then equating the determinant of the resulting system of equations to zero. If a single mode is considered in each section, the resulting equation becomes:

$$\tan\beta\ell = \frac{j\langle \hat{E},\hat{e}\rangle \langle \hat{H},\hat{h}\rangle \sinh \bar{\gamma}(L-\ell)}{\langle \hat{E},\hat{e}\rangle^2 \cosh^2 \bar{\gamma}\left(\frac{L-\ell}{2}\right) - \langle \hat{H},\hat{h}\rangle^2 \sinh^2 \bar{\gamma}\left(\frac{L-\ell}{2}\right)}$$

Results

Properties of the hybrid modes in an infinite waveguide are explored by first investigating the characteristic equation (4). The first few roots of this equation for a typical set of parameters are given in Table 1. As expected, the values of these roots were verified to be relatively insensitive to the radius b of the metallic waveguide enclosure, since the fields are almost entirely concentrated inside the dielectric, and only evanescent fields exist outside. This is also illustrated in the ω-β diagram of Figure 2 for the hybrid HE_{11} mode. Figure 3 is a mode chart for a dielectric loaded cavity of the same parameters given in Table 1. The plot of $(fD)^2$ versus $(D/L)^2$, where D = 2a is the diameter of the dielectric, is almost a straight line, similar to the case of the homogeneously filled waveguide.

Table 1. Roots of the Characteristic Equation for Hybrid Modes
a = 0.394", b = 0.5", f = 4.0 GHz, $\varepsilon_{r_1} = 37.6$, $\varepsilon_{r_2} = 1.0$

Mode	Root	Mode	Root
HE_{11}	2.2607	HE_{31}	6.7949
HE_{21}	3.6865	HE_{14}	7.1294
HE_{12}	4.4053	HE_{32}	8.0237
HE_{13}	5.3516	HE_{24}	8.2617
HE_{22}	5.8555	HE_{15}	8.5313
HE_{23}	6.6963	HE_{33}	9.3281

A comparison of measured results of the resonant frequencies of several resonators given in Reference 2 with the computed values using equations (4) and (5) is given in Table 2. The computed values agree remarkably with the measurements.

Table 2. Computed and Measured Resonant Frequencies for Several Resonators

a"	b"	ℓ"	L"	ε_{r_1}	Frequency (Measured) MHz	Frequency (Computed) MHz
.394	.5	.315	.6	37.6	3368	3371
.341	.5	.319	.83	37.25	3928	3930
.315	.5	.272	.56	37.6	4196	4192
.268	.5	.22	.48	38.2	4994	5001

Conclusions

The method presented for the computation of the resonant frequencies of dielectric loaded waveguide cavities is shown to be capable of providing quite accurate results. The method is applicable for non-axially symmetric modes and can be used to construct "mode charts" to predict both "desired" and "spurious" modes. An example of such a mode chart is given, and shown to be similar in shape to the homogeneously filled cavities.

References

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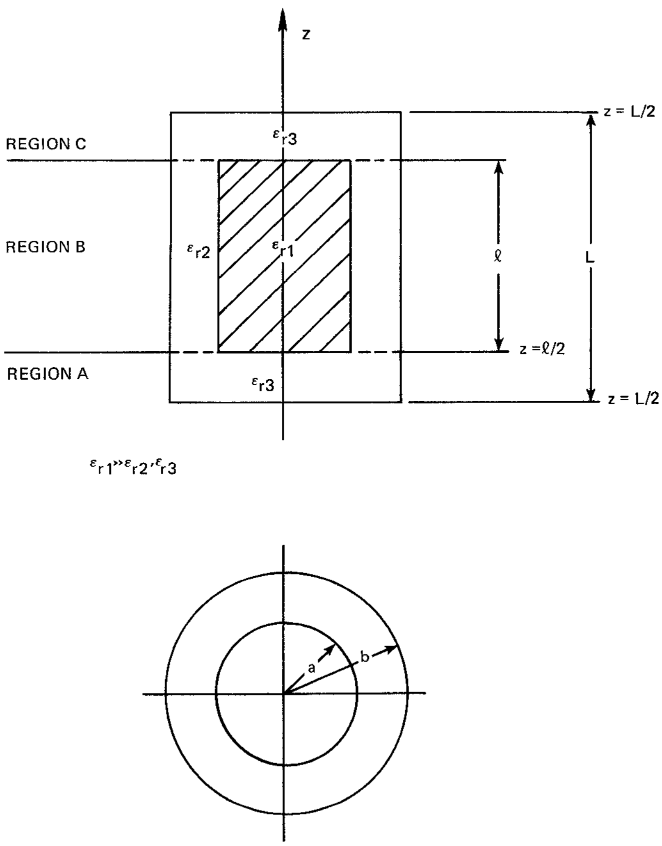

Figure 1. Dielectric Loaded Circular Cavity

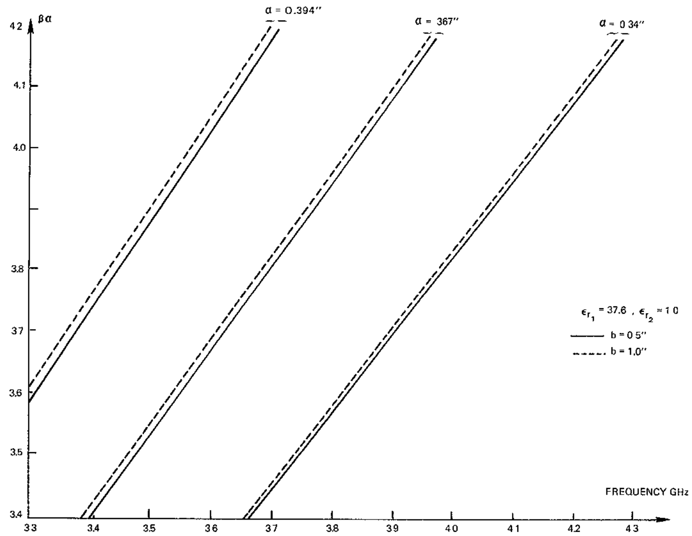

Figure 2. Omega-Beta Diagrams for the HE_{11} Mode

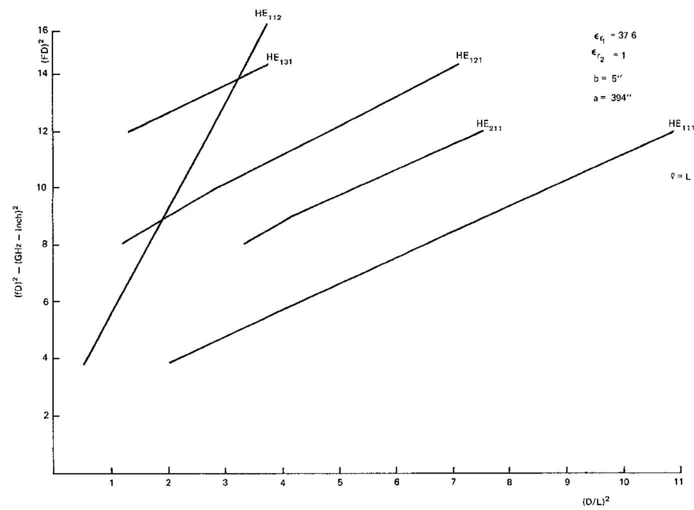

Figure 3. Mode Chart for a Dielectric Loaded Waveguide Resonator